Multi-angle Simulation of Neutrino Flavor Transformation in Supernovae

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Outline

- The problem of Supernova Neutrino Flavor Transformation
  - The essentials of Neutrino mixing.
  - Core collapse from a Neutrinos point of view.
  - Handling the full 3X 3 non-linear and geometry dependent problem.
- What have we learned so far?
  - Geometry cannot be ignored.
  - Neutrino self coupling has some strange effects.
- What needs to done?
  - Look at more kinds of Supernovae, and more evolutionary epochs.
  - Drill more holes to see how neutrino self coupling behaves.
  - Develop a better physical lay of the land

Neutrino Mass: what we know and don't know

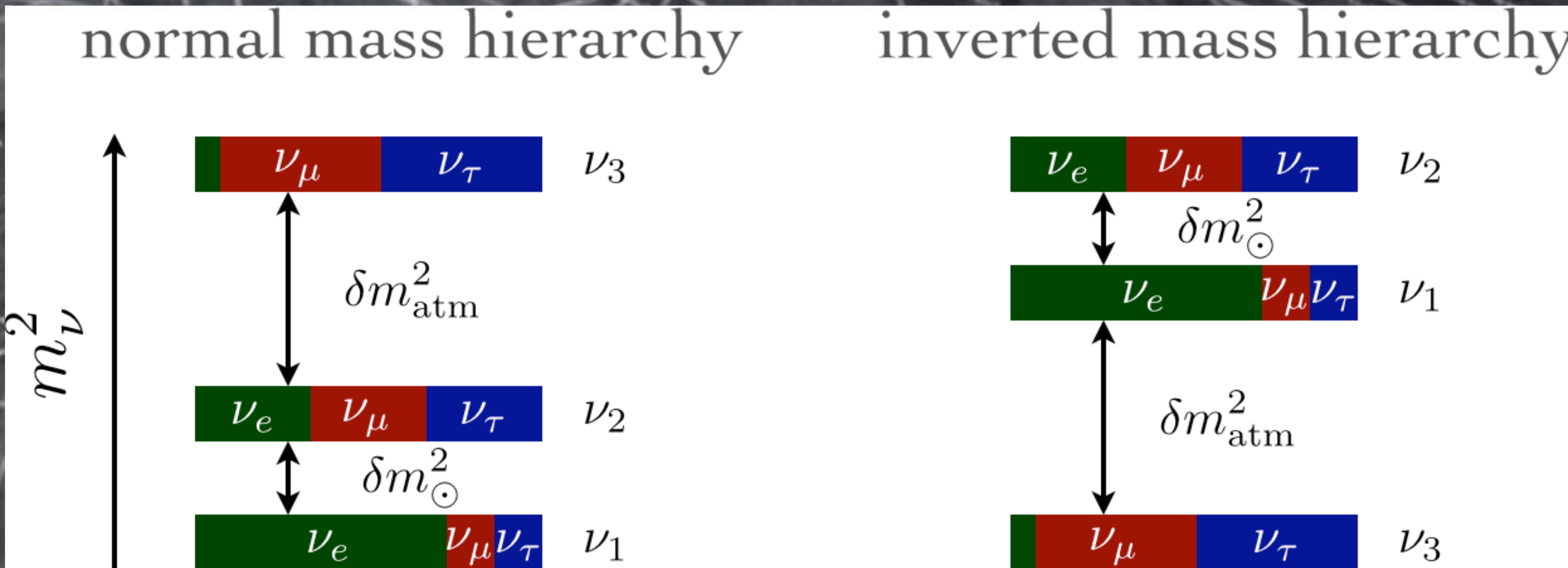
We know the *mass-squared* differences:

$$\delta m^2_{\odot} \approx 8 \times 10^{-5}\,\mathrm{eV}^2$$

$$\delta m^2_{\mathrm{atm}} \approx 3 \times 10^{-3}\,\mathrm{eV}^2$$

$$e.g., \quad \delta m^2_{21} \equiv m_2^2 - m_1^2$$

We *do not* know the *absolute masses* or the *mass hierarchy*:



Neutrino Mixing: How do we associate flavor states to mass states?

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U_m \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

$$U_m = U_{23} U_{13} U_{12}$$

$$U_{23} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

$$U_{13} \equiv \begin{pmatrix} \cos\theta_{13} & 0 & e^{i\delta}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix}$$

$$U_{12} \equiv \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sin^2 2\theta_{23} \approx 1.0$$

$$\tan^2 \theta_{12} \approx 0.42 \leftrightarrow 0.45$$

$$\sin^2 2\theta_{13} < 0.1$$

4 parameters

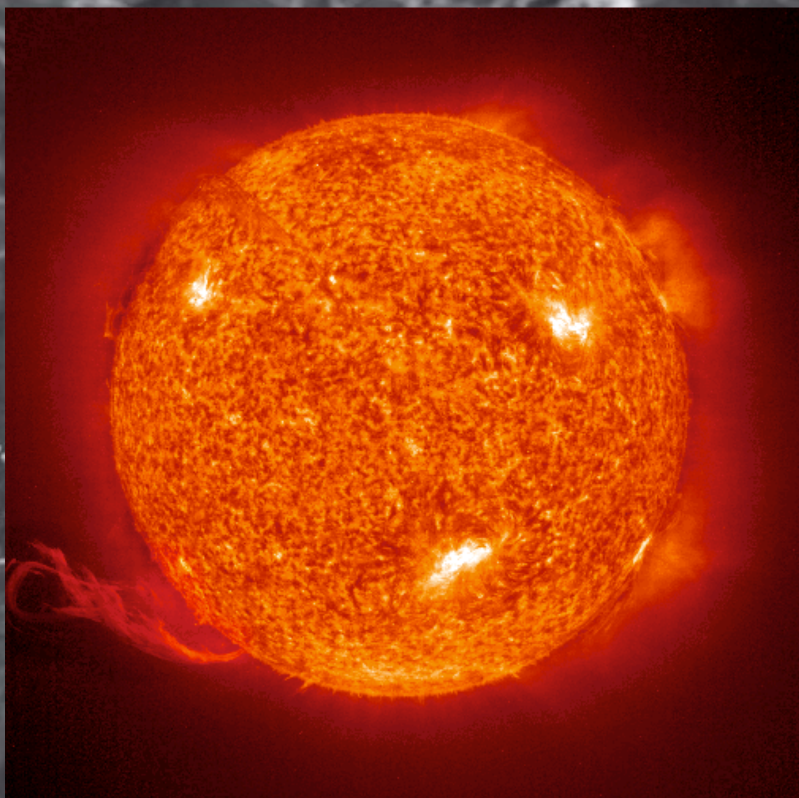
we know θ_{12} & θ_{23}

we need δ & θ_{13}

Supernova!

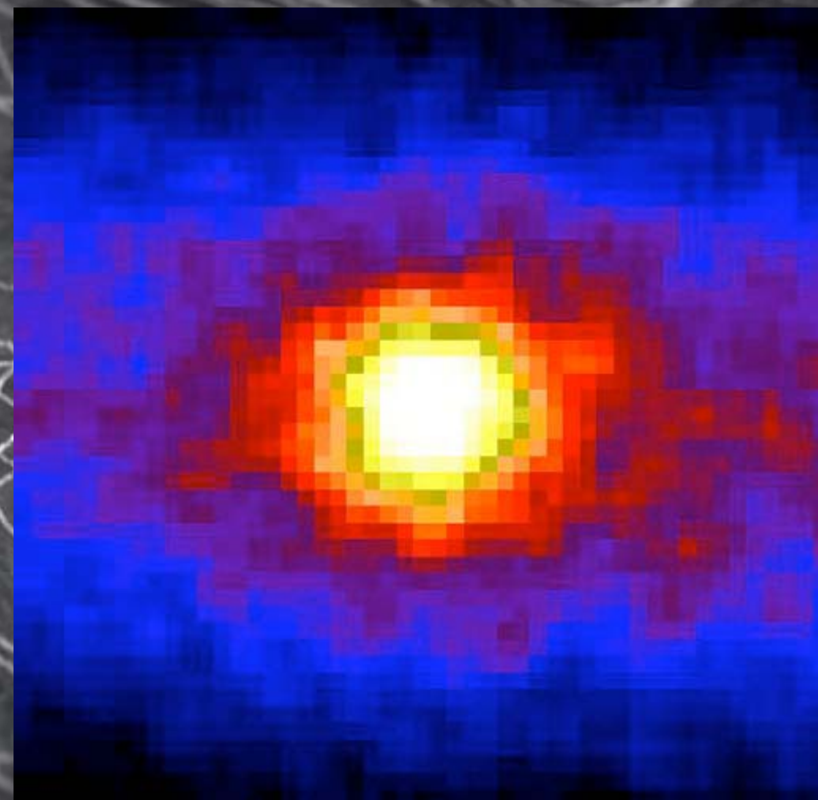
- Whopping bright sources of neutrinos.
  - Neutrino Luminosities of 10^{51} to 10^{54} erg/s
  - May be the only way to detect some core collapses!
- Neutrinos are the only method to see way down inside of an exploding star.
- Important locations for determination of fundamental Neutrino properties.

Photons vs. Neutrinos



The sun in x-rays (photons)

(Cornell)



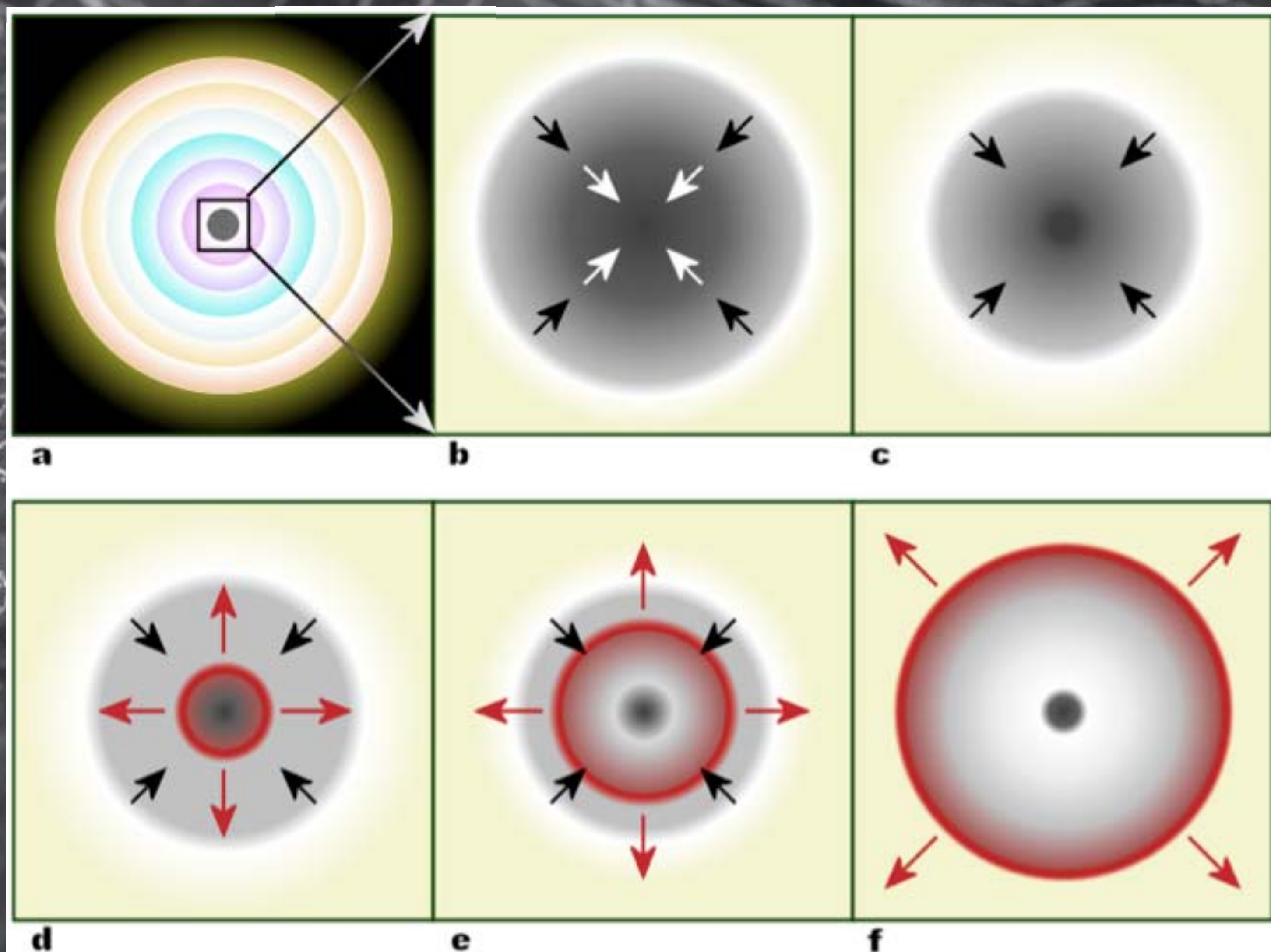
the sun in neutrinos

-composite from SuperK

(R. Svoboda, LBL)

Core Collapse Supernova

Iron Core ... Collapses... Halts at Nucl. Density...

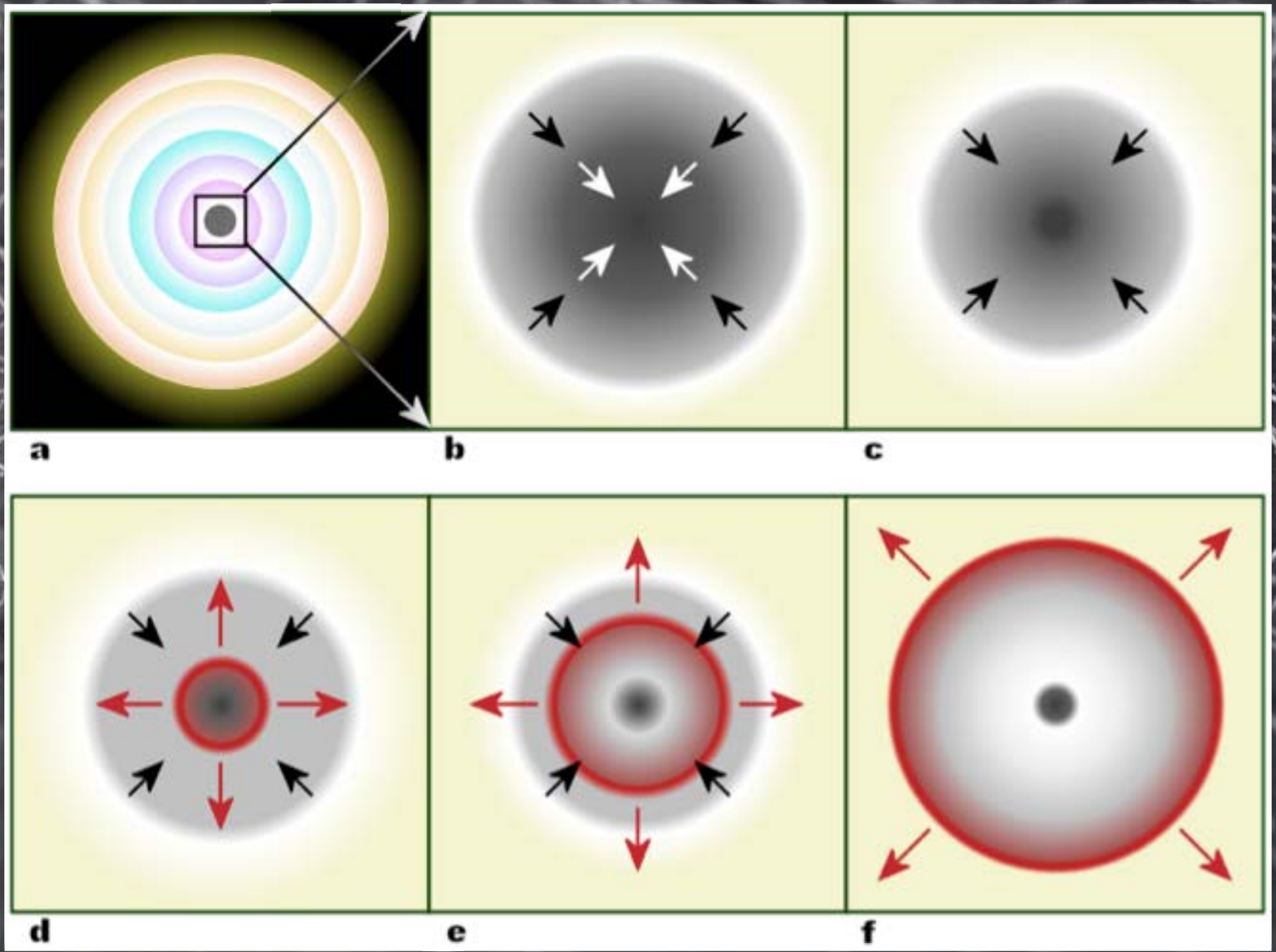


Shock Wave Launched... Stalls... Is Reheated and Explodes.

Wikipedia: Supernova

From a Neutrinos Perspective

Iron Core ... Neutrinos Trapped... And Thermalized...



Neutrino Sphere Shines... Deleptonizes... Freely Streams Through Envelope.

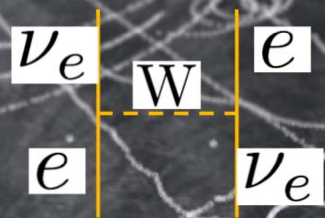
Wikipedia: Supernova

Coherent Flavor Evolution for Neutrino i

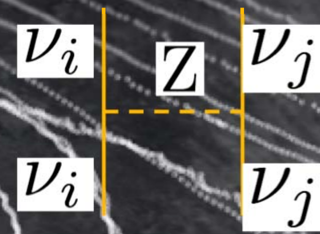
$$\psi_{\nu,i} = \begin{bmatrix} \text{amplitude to be } \nu_{\mathrm{e}} \\ \text{amplitude to be } \nu_{\mu,\tau} \end{bmatrix}$$

$$i\,\frac{\partial}{\partial t}\psi_{\nu,i} \;=\; \left(\mathcal{H}_{\mathrm{vac},i} + \mathcal{H}_{\mathrm{e},i} + \mathcal{H}_{\nu\nu,i}\right)\psi_{\nu,i}$$

neutrino-electron
charged current
forward exchange
scattering



neutrino-neutrino
neutral current
forward scattering



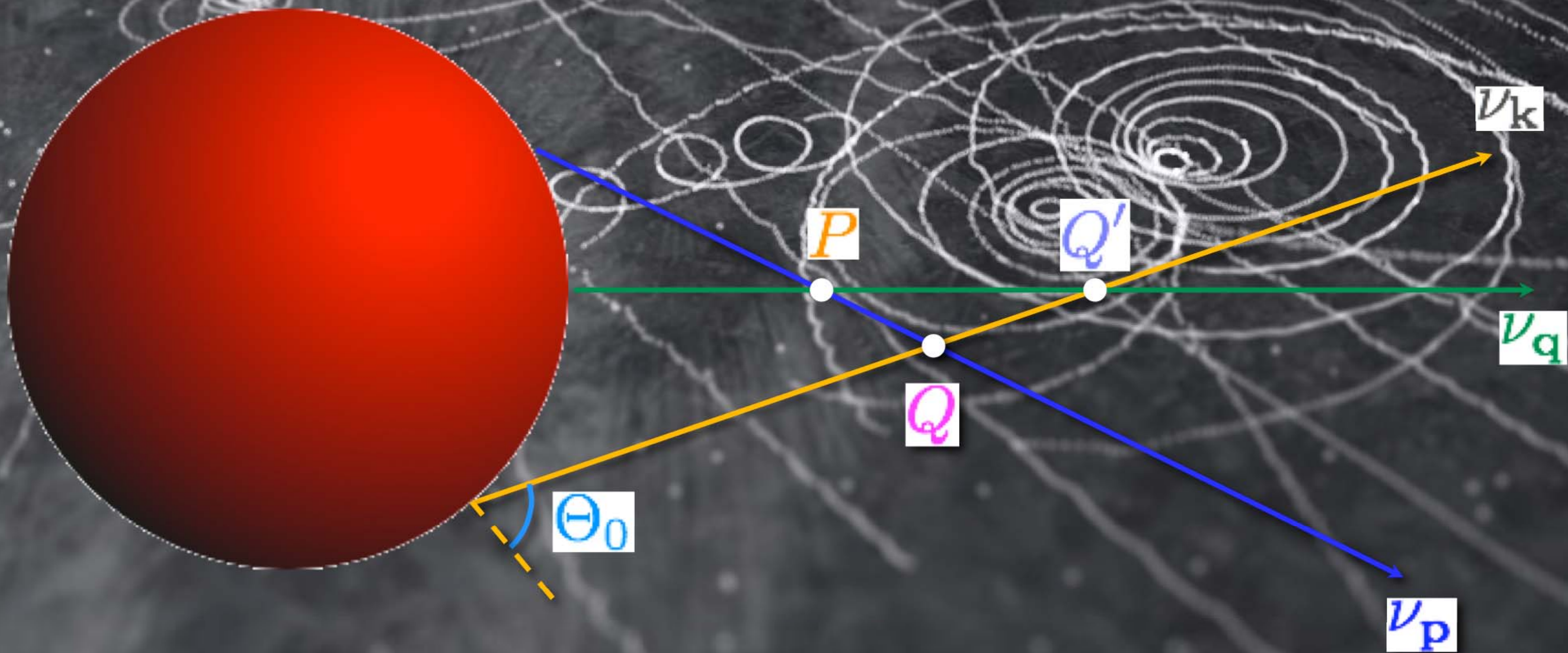
Neutrino Self Coupling - the source of nonlinearity

$$\mathcal{H}_{\nu\nu,i} \equiv \sqrt{2}G_{\mathrm{F}}\sum_j \left(1-\hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_j\right) n_{\nu,j}\,\psi_{\nu,j}\psi^{\dagger}_{\nu,j}$$

$$-\sqrt{2}G_{\mathrm{F}}\sum_j \left(1-\hat{\mathbf{k}}_i\cdot\hat{\mathbf{k}}_j\right) n_{\bar{\nu},j}\,\psi_{\bar{\nu},j}\psi^{\dagger}_{\bar{\nu},j}$$

Neutrino Self-Coupling Represented as a Picture:

- Anisotropic, nonlinear quantum coupling of all neutrino flavor evolution histories



The Codes that Treat this Problem

- There are two codes that are used to model neutrino flavor evolution: FLAT (UCSD), BULB (LANL)
- Both developed in parallel using different architectures. Validation was accomplished by comparing the two to each other, otherwise know as the boot-strap method.

How does Bulb work?

1. Start with initial conditions: Luminosity, Energy, NS radius, Mass Hierarchy, Mixing Angles, Angular resolution

2. Initialize Neutrinos in pure flavor states

3. Determine the size of the next radial step outward. Compute Vacuum and Matter contributions.

4. At the initial position, calculate the intersection of each angular bin with every other angular bin.

5. Use that information to find the number density for each energy-angle bin, seen by every other energy-angle bin.

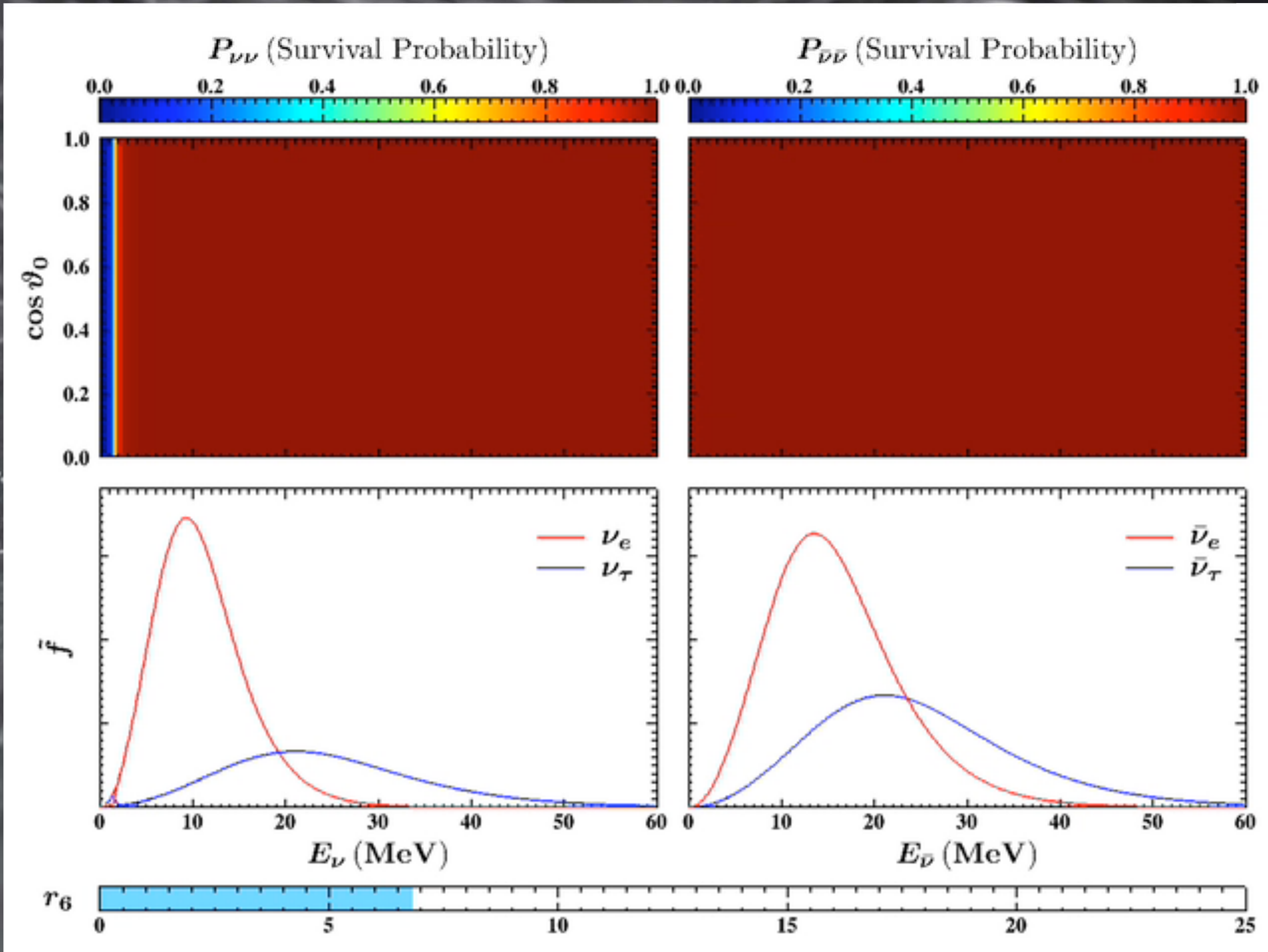
6. Use the current flavor state of each energy-angle bin to compute the neutrino self-coupling contribution.

7. Evolve the neutrino wave functions according to the pseudo-Schrodinger equation.

8. Repeat steps 3-7 twice, using half the radial step size.

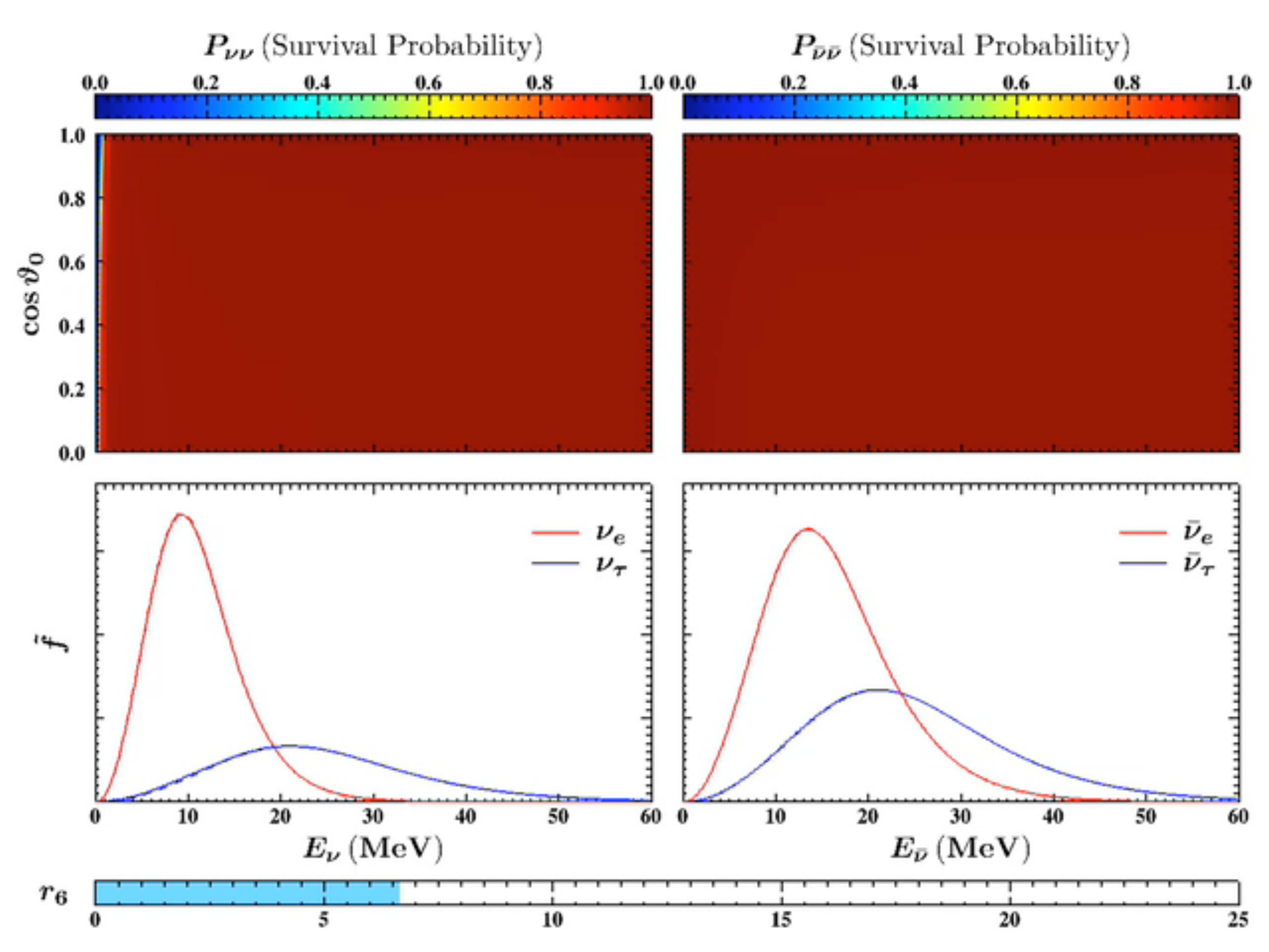
9. Check that both full step and half step flavor states agree to within preset error tolerance.

Late Time Core Collapse, Normal Mass Hierarchy, no Neutrino Self Coupling



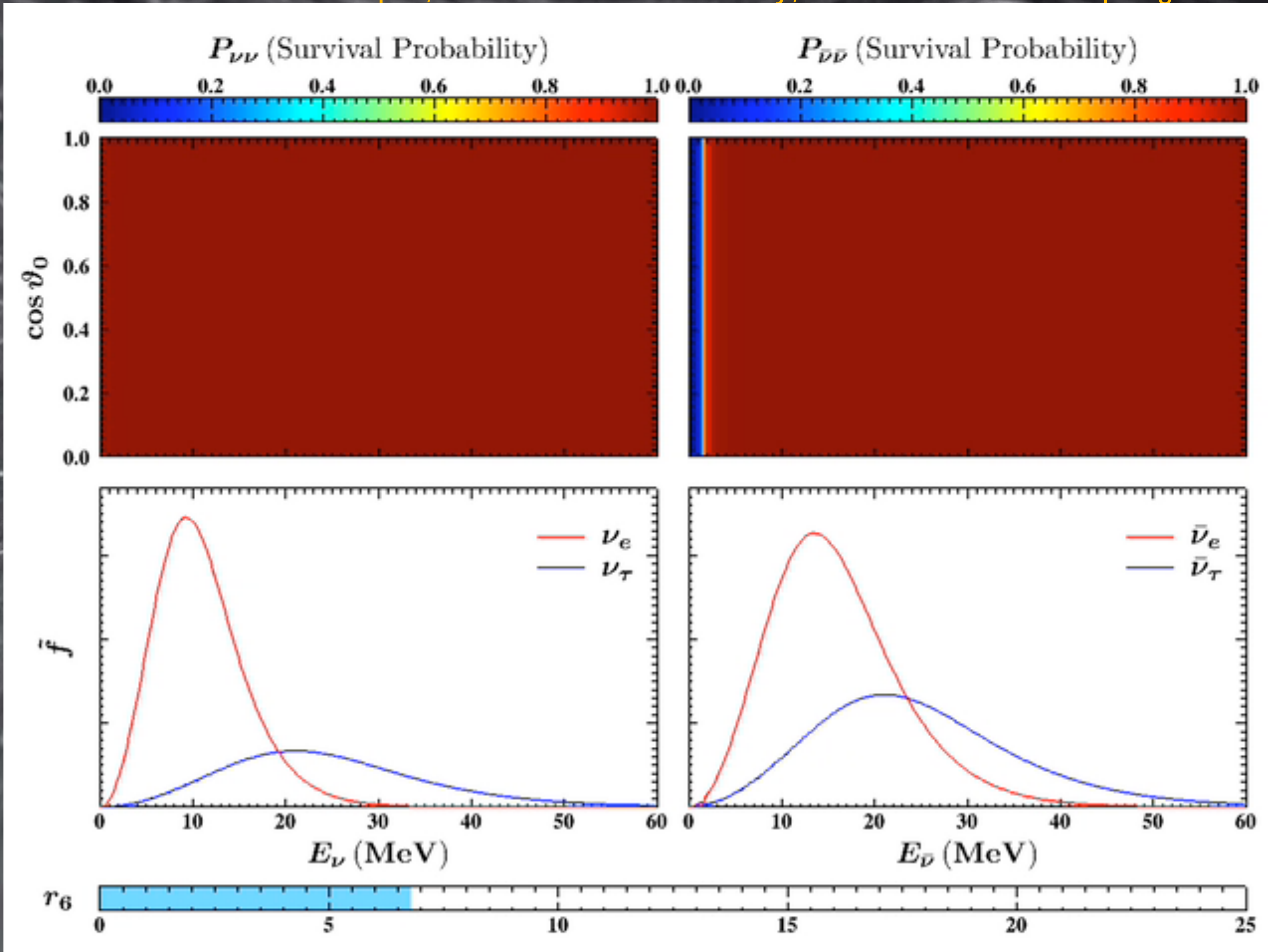
H. Duan, G. M. Fuller, J. Carlson, 2008 Comput. Sci. Disc. 015007

Late Time Core Collapse, Normal Mass Hierarchy, with Neutrino Self Coupling



H. Duan, G. M. Fuller, J. Carlson, 2008 Comput. Sci. Disc. 015007

Late Time Core Collapse, Inverted Mass Hierarchy, no Neutrino Self Coupling



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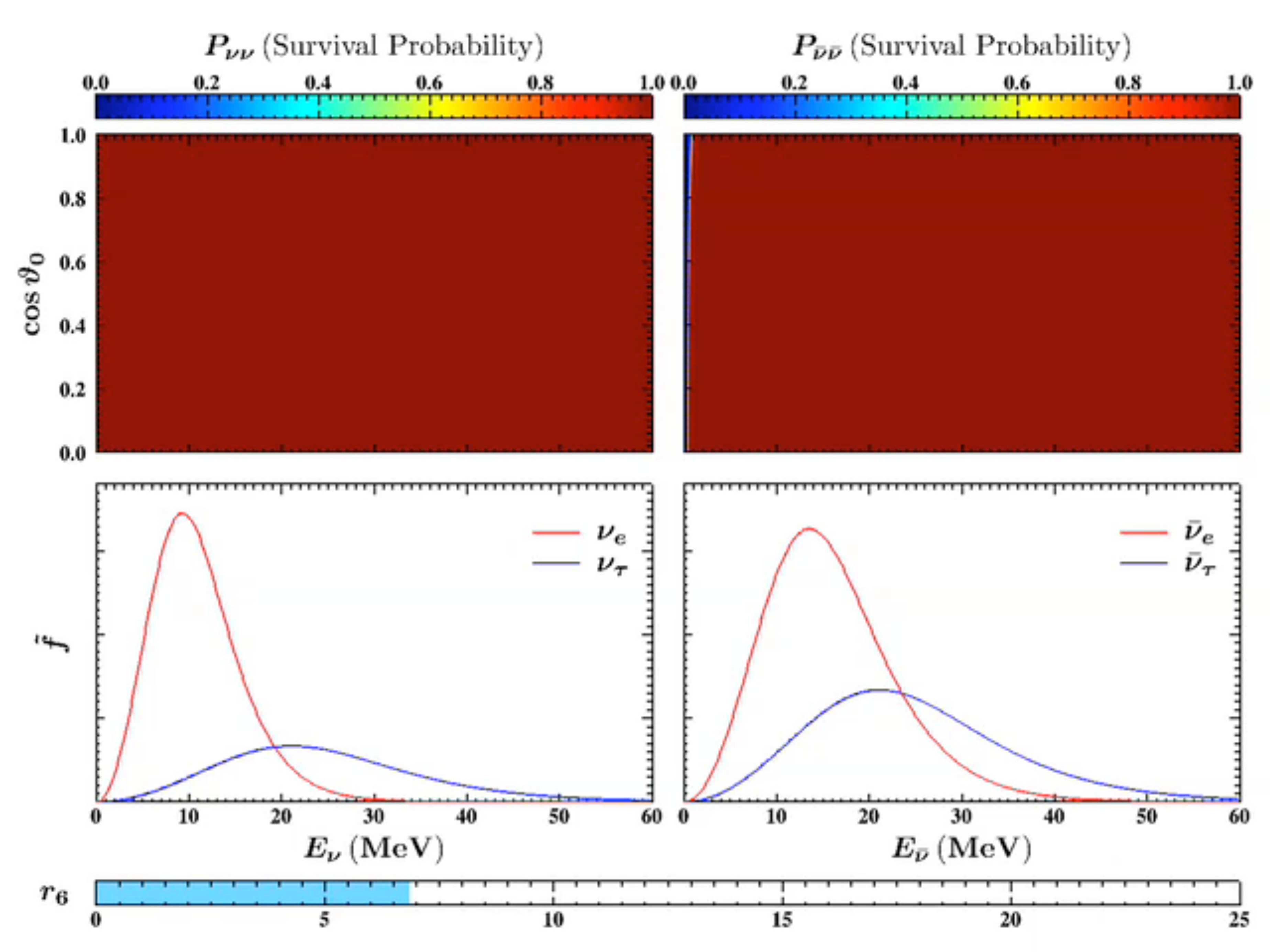
Late Time Core Collapse, Inverted Mass Hierarchy, with Neutrino Self Coupling



H. Duan, G. M. Fuller, J. Carlson, 2008 Comput. Sci. Disc. 015007

Collective Neutrino Oscillation

Neutrino Flavor Isospin:

$$\omega = \begin{cases} +\dfrac{\delta m^2}{2E} & \text{for neutrinos} \\ -\dfrac{\delta m^2}{2E} & \text{for antineutrinos} \end{cases}$$

$$\begin{bmatrix} e^- \\ \nu_e \end{bmatrix} \begin{bmatrix} \mu/\tau \\ \nu_\mu/\nu_\tau \end{bmatrix}$$

	e- flavor	τ' flavor	mixed
$\vec{s} \equiv \psi^{c.t.}_{\nu,E} \dfrac{\vec{\sigma}}{2} \psi_{\nu,E}$	$\uparrow$	$\downarrow$	$\rightarrow$

Reorganizing the Mean Field Equation

$$\frac{d}{dt}\vec{s}_\omega = \vec{s}_\omega \times \vec{H}_\omega$$

$$\vec{H}_\omega = \vec{H}_{vac} + \vec{H}_{matter} + \vec{H}_{\nu\nu}$$

$$\vec{H}_{vac} = \omega \hat{e}_z^V$$

$$\vec{H}_{matt} = -\sqrt{2} G_F n_e \hat{e}_z^f$$

$$\vec{H}_{\nu\nu} = -2\sqrt{2} G_F n_\nu^{tot} \int_{-\infty}^{\infty} d\omega' f_{\omega'} \vec{s}_{\omega'} = -\mu \langle \vec{s} \rangle$$

Regular Precession Ansatz

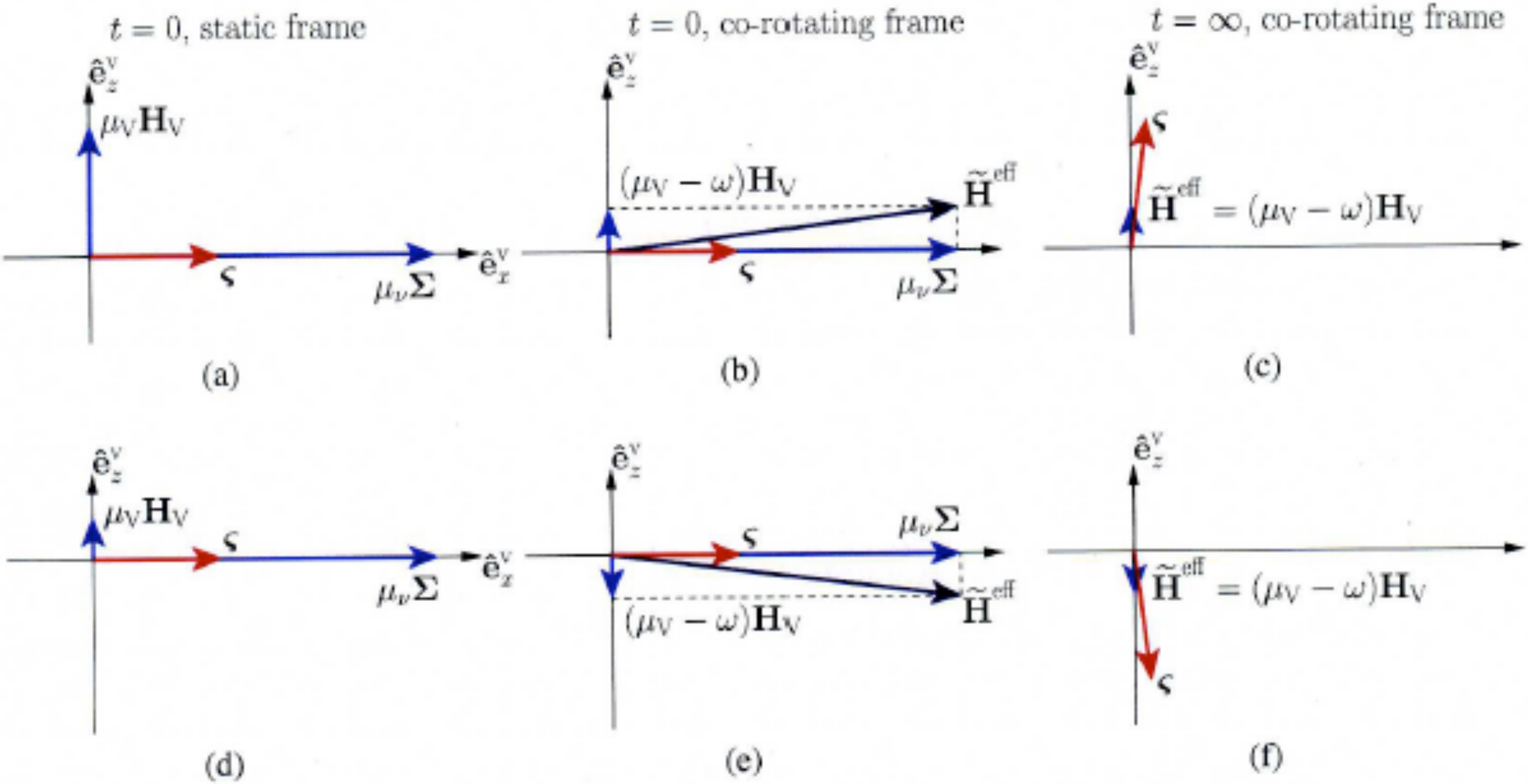
$$\vec{s}_\omega \times \tilde{\vec{H}}_\omega = 0$$

$$\frac{d}{dt}\vec{s}_\omega = \vec{s}_\omega \times \omega_{pr}\hat{e}_z^V$$

$$1 = -\frac{1}{2}\int_{-\infty}^{\infty} d\omega \frac{\varepsilon_\omega f_\omega}{\sqrt{\left[\left(\omega-\omega_{pr}\right)/\mu - \langle\vec{s}_z\rangle\right]^2 + \langle\vec{s}_\perp\rangle^2}}$$

$$\omega_{pr} = -\frac{1}{2}\int_{-\infty}^{\infty} d\omega \frac{\omega\varepsilon_\omega f_\omega}{\sqrt{\left[\left(\omega-\omega_{pr}\right)/\mu - \langle\vec{s}_z\rangle\right]^2 + \langle\vec{s}_\perp\rangle^2}}$$

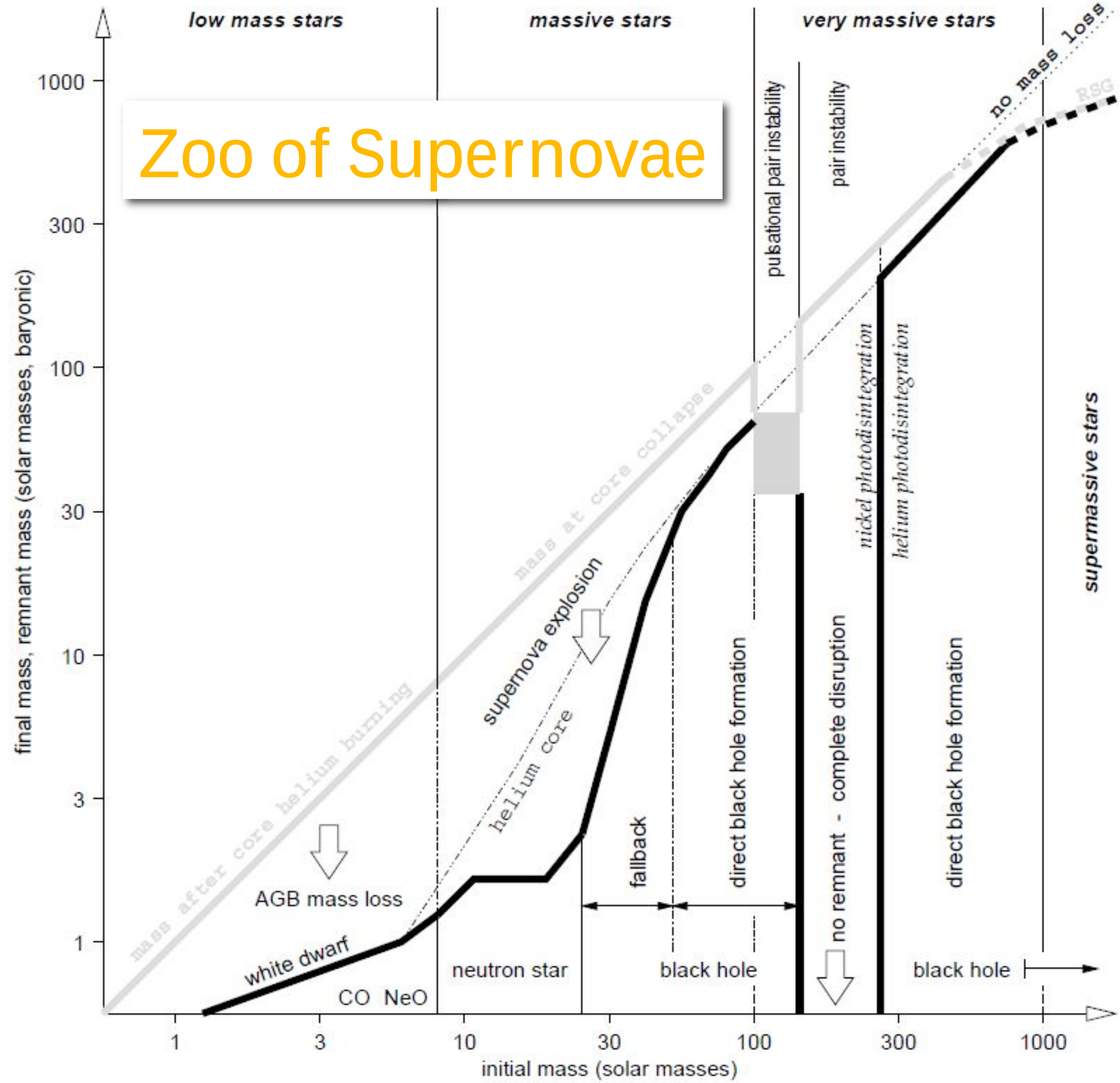
Spectral Split



That was Interesting!

- Geometric effects played a significant role in the resulting spectra.
- Non-linear self coupling produced collective Neutrino flavor transformation.
- This simulation was just one special case!

Zoo of Supernovae



Heger, Woosley, et. al., astro-ph/0112059, 2001

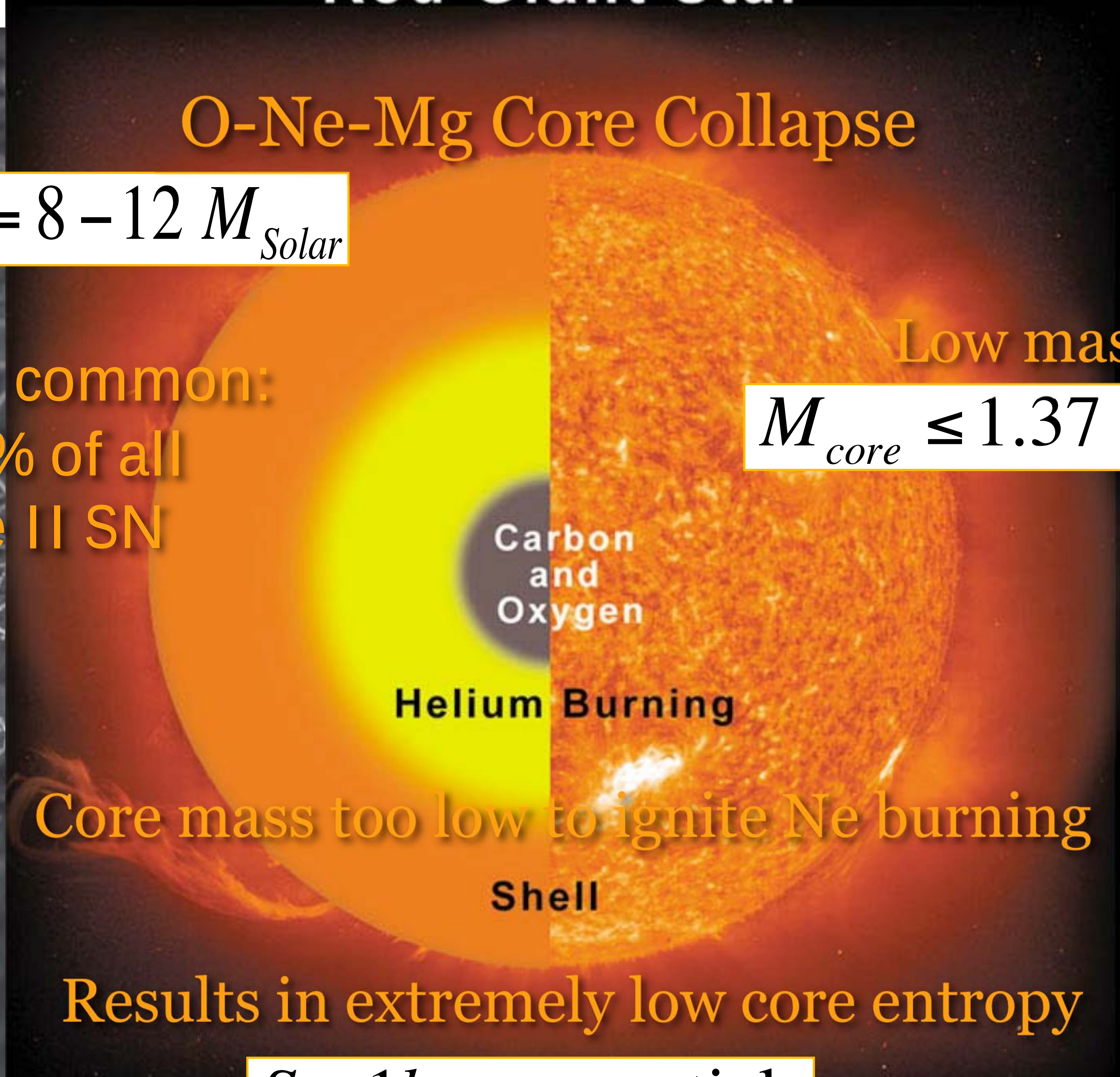
O-Ne-Mg Core Collapse

$M = 8 - 12\ M_{Solar}$

Very common:
~20% of all
Type II SN

Low mass core

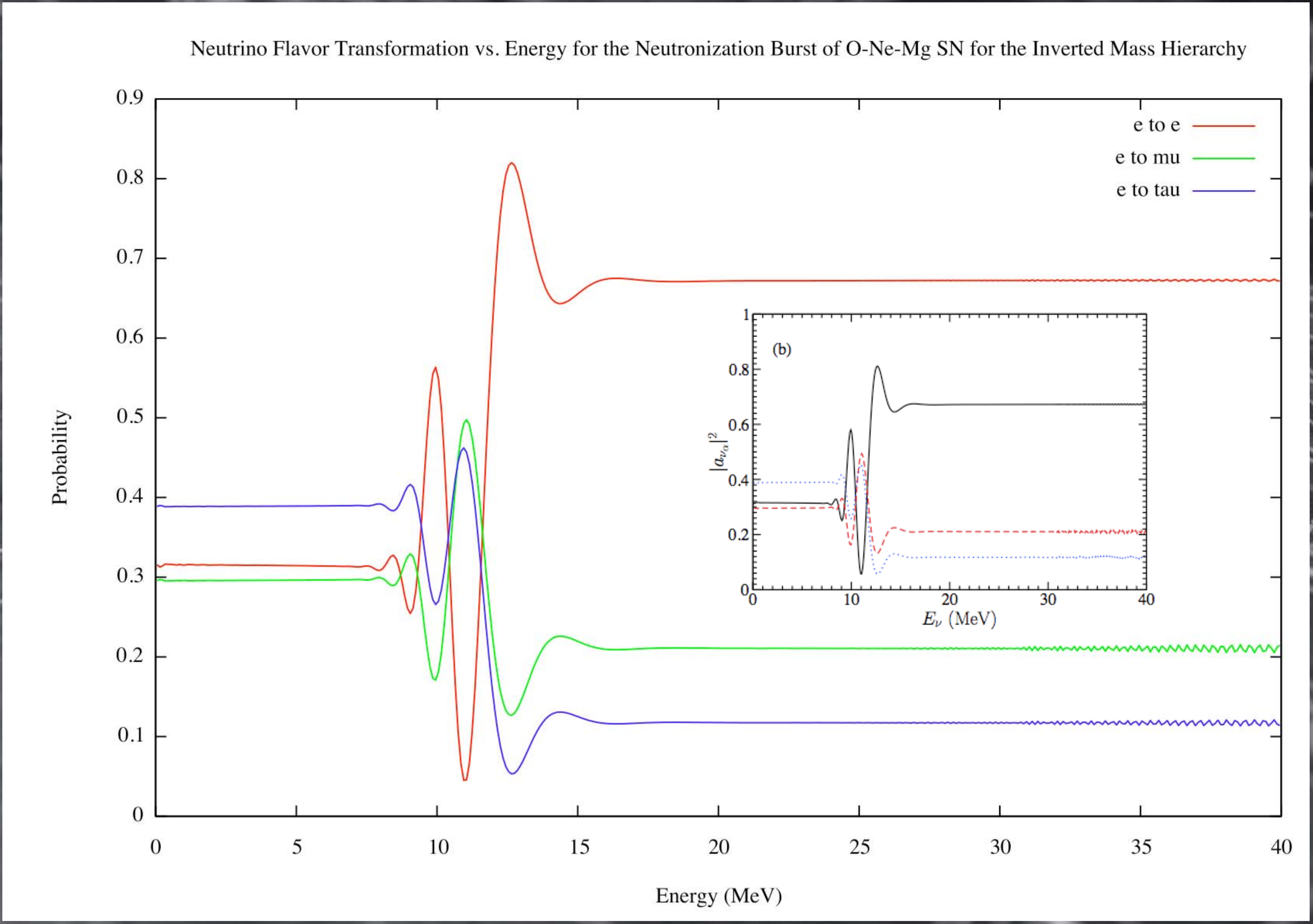
$M_{core} \leq 1.37\ \ M_{Solar}$



Core mass too low to ignite Ne burning

Results in extremely low core entropy

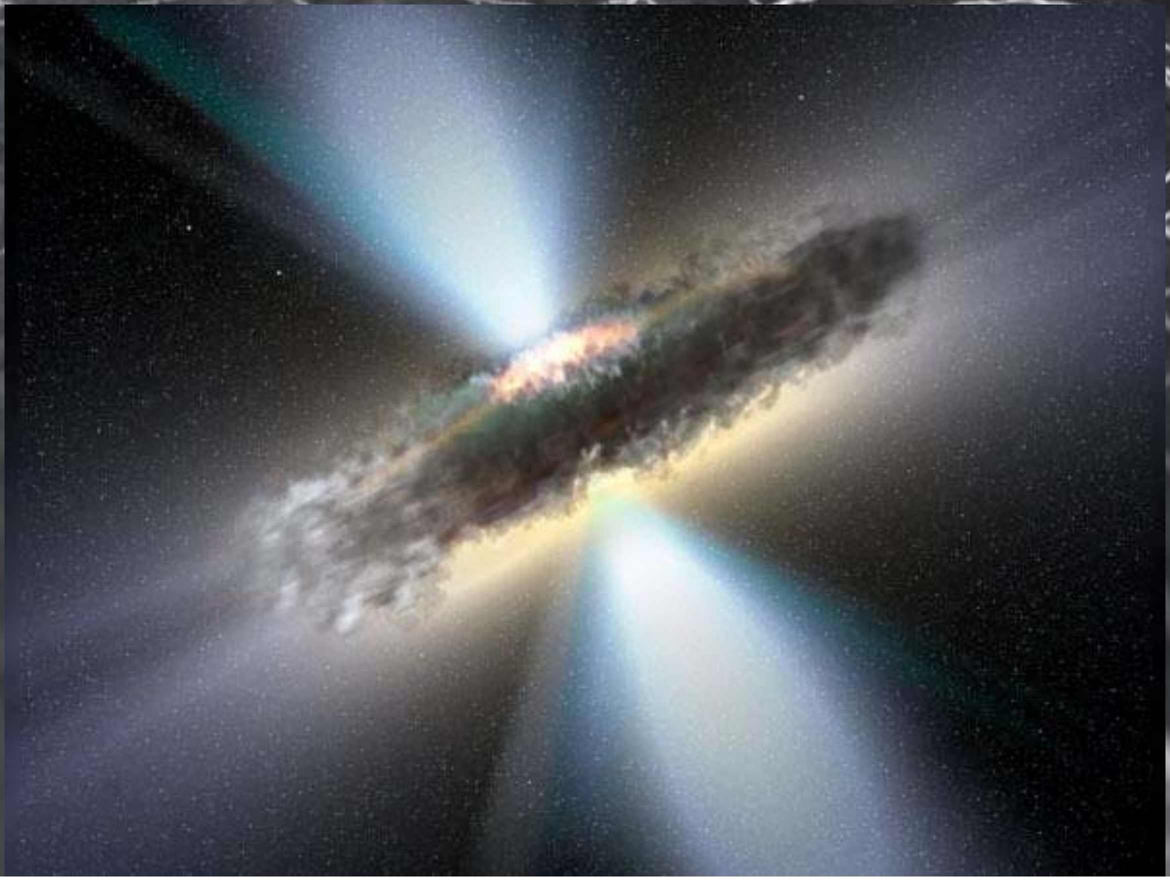
$S \approx 1 k_B$ per particle



H. Duan, G. M. Fuller, J. Carlson, Y. Z. Qian, Phys. Rev. Let. 100 021101 (2008).

Other Applications

In principle, the current codes can be adapted to any axis-symmetric geometry:



In Review

- Examining flavor transformation in various types of supernova environments will build a robust framework for detecting and analyzing those events.
- Developing the code to handle more generalized scenarios will broaden the ways in which we can examine flavor transformation.
- Poking these holes through the veil of nonlinearity will help to create a comprehensive, physically grounded picture of neutrino flavor transformation.

References

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Pair Instability Supernovae

Extremely Massive

$$M \geq 100\, M_{Solar}$$

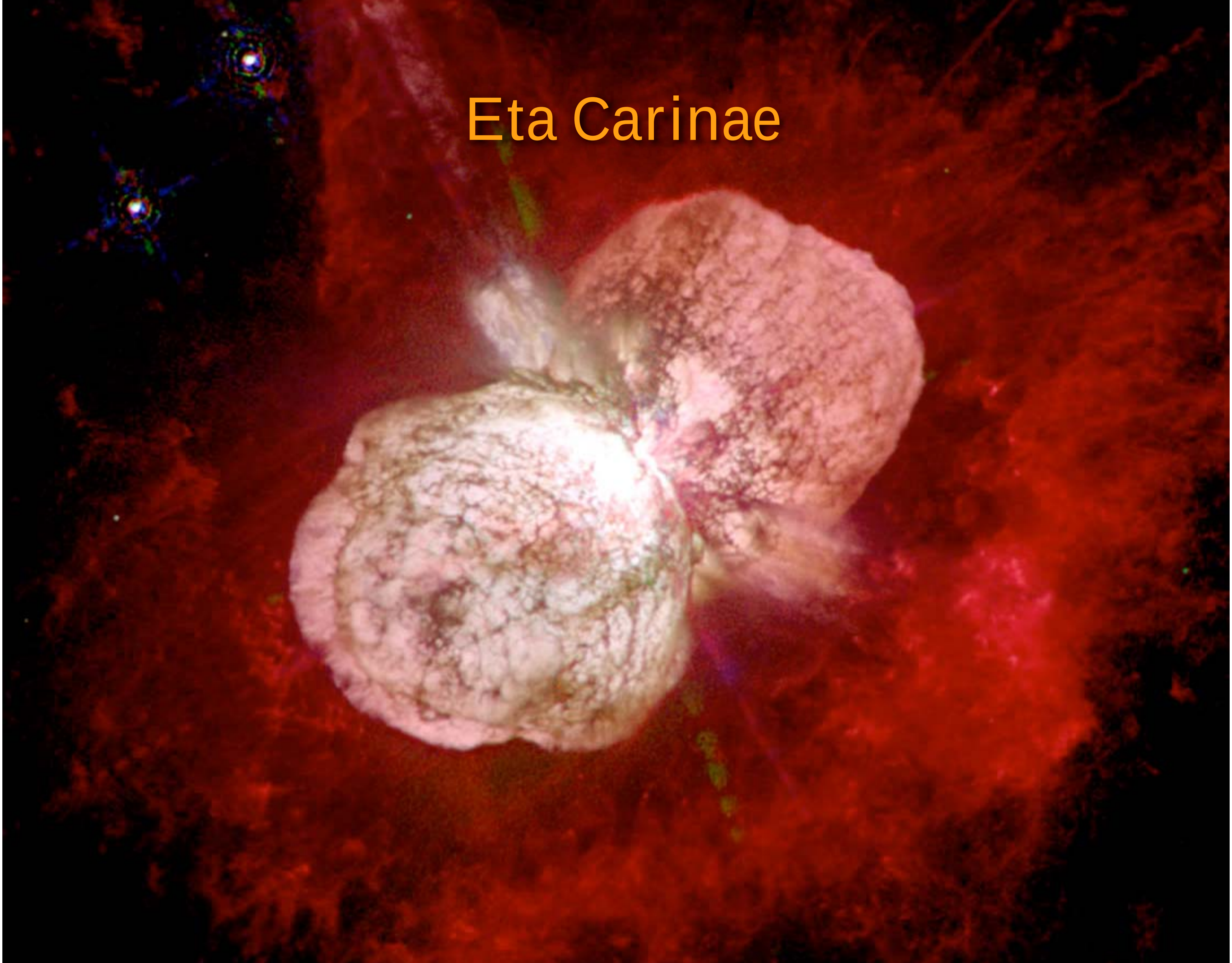
High Enthalpy PerBaryon

Supported by radiation pressure!

As T approaches 1 MeV, an interesting thing occurs

$$2\gamma \rightarrow e^- + e^+$$

Eta Carinae



Start with the Lagrangian Densities for the Vaccuum and Matter Interactions

$$L_0 = \overline{\nu}\left(i\not{\partial} - M\right)\nu$$

$$L_W = -\left(G_F/\sqrt{2}\right) j^{\mu}\overline{\nu}\gamma_{\mu}\left(1+\gamma_5\right)N\nu$$

Where:

$$j^{\mu} = \overline{e}\gamma^{\mu}\left(1+\gamma_5\right)e$$

$$N = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

This Gives the Field Equation

$$\left(i\not\partial - M\right)\nu = \left(G_F/\sqrt{2}\right) j^{\mu}\gamma_{\mu}\left(1+\gamma_5\right)\nu$$

Taking:

$$\nu_{L/R} = (1 \pm \gamma_5)\nu/2$$

$$i\not\partial\nu_L - M\nu_R = \sqrt{2}G_F j_{\mu}\gamma^{\mu} N \nu_L$$

$$i\not\partial\nu_R - M\nu_L = 0$$

Eliminating the Right Handed Field

$$\left(\partial^{\mu}\partial_{\mu}+M^{2}\right)\nu_{L}=-i\partial\!\!\!/\left(\sqrt{2}G_{F}j_{\mu}\gamma^{\nu}N\nu_{L}\right)$$

We need to make a simplifying assumption:

$$j_{\mu}=\rho(x)\delta_{\mu 0}$$

Which leads to solutions of the form:

$$\left|\nu(x,t)\right\rangle=\left|L(x)\right\rangle e^{-iEt}$$

This can be reduced to a time independent equation

$$\left(E^2 - M^2 + \nabla^2\right)L(x) = \sqrt{2}G_F\left[E - \left(-i\gamma_0\vec{\gamma} \bullet \vec{\nabla}\right)\right]\rho N L(x)$$

For relativistic Neutrinos: $\gamma^0\vec{\gamma} \bullet \left(-i\vec{\nabla}\right)L(x) = -EL(x)$

Leading to a mean field expression:

$$\left(F + \nabla^2\right)L(x) = 0$$

Where: $F = E^2 - M^2 - 2\sqrt{2}G_F\rho E N$

For Non-Uniform Matter

$$\left[\sqrt{F} - i\frac{d}{dx}\right]\left[\sqrt{F} + i\frac{d}{dx}\right]L(x) = 0$$

We can expand to lowest order in M^2/E^2 and $G\rho E/E^2$:

$$\sqrt{F} = E - \frac{M^2}{2E} - \sqrt{2}G_F\rho N$$

Now we invoke coherent forward scattering

$$\left(E - \frac{M^2}{2E} - \sqrt{2}G_F\rho N + i\frac{d}{dx}\right)L(x) = 0$$

All of the matrix components proportional to the Identity will not contribute to flavor evolution. Taking only the traceless components gives us our mean field equation.